Study Questions

1. You are studying mercury concentrations in a population of freshwater mussels. You want to have 75% power in your ability to detect a difference of 5 ppm between mercury concentrations in mussels and an EPA standard. Your significance level is 0.05. The variance in mussel mercury concentration obtained in a preliminary sample is 100 ppm^2. You have enough money to measure mercury concentration in 25 mussels. Will this be enough, or do you need to ask your boss for additional funds to collect a larger sample?

Method 1

$\alpha = 0.05$, $\beta = 0.25$, $\nu = 25 - 1 = 24$

$$n \geq \frac{s^2}{\delta^2}\left(t_{\alpha,\nu} + t_{\beta(1),\nu}\right)^2$$

$$n \geq \frac{100}{(5)^2}(2.064 + 0.685)^2$$

$$n \geq 4 * (2.749)^2$$

$$n \geq 30.2$$

so $n > 25$ ($31 > n > 25$)

Yes, do need to ask boss for more funds.

Method 2

$\alpha = 0.05$, $\beta = 0.25$, $\nu = 25 - 1 = 24$

$$t_{\beta(1),\nu} \leq \frac{\delta}{\sqrt{s^2/n}} - t_{a,\nu}$$

$$\leq \frac{5}{\sqrt{100/25}} - t_{0.05(2),38}$$

$$\leq 2.5 - 2.064$$

$$\leq 0.436$$

$$t_{0.25(1),24} = 0.685$$

so $\beta > 0.25$, so power < 0.75

Yes, do need to ask boss for more funds.

2. You are studying effects of air pollution levels on growth rates of Douglas fir trees. You measure growth rates at four different pollution levels, using sample sizes (number of replicates) of 2, 5, 9, and 15 trees. You do not obtain a significant effect of pollution level using single-factor analysis of variance. You suspect that there is a real effect of pollution level, but that your study design was not sufficiently powerful to detect it. What are two ways that you could increase power in a second study? (Do not change significance level, α.)

<u>Answer</u>: (1) Balance your design (equal sample sizes).
(2) Increase size of all samples, esp. smaller samples.

3 Your neighbor Jed wants to know if fuel economy using 89 octane gasoline differs from fuel economy using 87 octane gasoline. Jed knows that you are a statistical whiz, so he takes great care to design an unbiased test. Each day, Jed drives his van along the same route to and from work. He drives at the same speed, with the same tire pressure, with the same amount of weight in the van, … he even wears the same shoes. Jed's van has two gas tanks, one for 87 octane and the other for 89 octane. Jed switches from one tank to the other each day. On his way home from work, Jed fills the tank he has been using that day, calculates fuel economy for that day, and then switches to the other tank. After following this routine for fifty days (25 days with each kind of gas), Jed performs a two-sample t-test on his fuel economy data. ($H_0: \mu_1 = \mu_2$, where μ_1 is mean gas mileage with 89 octane, and μ_2 is mean gas mileage with 87 octane.) He obtains a t-value of 2.10, which exceeds the critical value of $t_{0.05(2),48}$. Jed declares that 89 octane gasoline yields different gas mileage than mileage obtained with 87 octane gasoline. You declare Jed's test to be invalid. You are correct – why?

> Answer: Jed's study used pseudoreplication. Jed repeatedly measured fuel used by the same vehicle, but treated those measurements as independent in his analysis. Jed's true sample sizes are $n_1 = n_2 = 1$, leaving him with zero degrees of freedom ($\nu = n_1 + n_2 - 2 = 0$). Cannot perform hypothesis test with zero degrees of freedom. If he wanted 48 degrees of freedom, he must measure fuel economy in samples of 25 vehicles each.
> A secondary issue is that Jed should have used a paired sample test, because his samples were paired by using the same vehicle in both samples. This issue alone would not make Jed's two-sample test invalid, just less powerful.
> Of course, Jed could use his measurements to make inferences about fuel economy with his van. That inference could not be extrapolated with confidence to fuel economy (89 octane vs. 87 octane) for vehicles in general.

For the following four questions (4 - 7), describe the following.
(a) The appropriate statistical analysis to perform, including number of tails, fixed or random effects, and parametric or nonparametric tests.
(b) State any assumptions necessary in using the appropriate statistical analysis.
(c) State the null hypothesis or hypotheses to be tested.
(d) State the criteria for rejection of the null hypothesis or hypotheses.

4 Research question: Does the density of barnacles in patches of rocky intertidal habitat at Larrabee State Park affect the number of predatory snails found there? Snails were counted in ten quadrats each at four levels of barnacle density. Snail abundances appear to be normally distributed. Use a significance level of $\alpha = 0.10$.

Answer: (a) Single factor analysis of variance. Two tails, random factor, parametric test if (b) satisfied; otherwise, use Kruskal-Wallis test.

> (b) Samples from normally distributed populations; equal variances.
> (c) H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_3$
> (d) $F_{calc} \geq F_{\alpha(1),3,36}$ e.g., if α = 0.10, $F \approx 2.25$ (use $F_{0.10(1),3,35} = 2.25$)
> (no value in table for 36 df, so use 35 df instead)

5 Analysis in problem 4 did show an effect of barnacle density on number of snails. At which levels of barnacle density does snail number differ? Use a significance level of $\alpha = 0.10$.

Answer: (a) Multiple comparison test (e.g., Tukey test). Two tails, parametric test.
(b) Samples from normally distributed populations; equal variances.
(c) H_0: $\mu_A = \mu_B$, where A,B $\in$ {crevices, overhangs, horizontal, vertical surfaces}
(d) $q_{calc} \geq q_{\alpha(2),36,4}$ e.g., if $\alpha = 0.10$, $q \approx 3.349$ (use $q_{0.10,4,30} = 3.386$)
(no value in table for 36 df, so interpolate, or use 30 df instead q_crit = 3.386)

6 How would you address problem 4 if snail abundances deviate severely from normality?

Answer: (Ignoring possibility of transforming data to normality) Perform a Kruskal-Wallis test (nonparametric single-factor analysis of variance).

7 Research question: Do parasitoid wasps affect variability in abundances of their moth prey? Moth abundances were recorded in twelve experimental plots, four replicates at each of three wasp densities (none, low, high). Use a significance level of $\alpha = 0.05$.

(a) Bartlett's test for homogeneity of variances.
Two tails; parametric test

(b) Moth abundances (or rather their residuals) normally distributed.

(c) $H_0 : \sigma_1^2 = \sigma_2^2 = \sigma_3^2$

(d) $B_{c\ calc} \geq \chi^2_{\alpha,k-1}$, where $B_c = B/C$
Here, k=3 and α=0.05, so $\chi^2_{\alpha,k-1} = \chi^2_{0.05,2} = 5.991.$

8 Research question: Does nest productivity (number of fledglings produced) of songbird nests differ among nests in forests, wetlands, and grasslands? A significance level of 0.05 was selected. Productivity was recorded from a random sample of ten nests in each of the three kinds of habitats (30 nests total). Nest productivities were normally distributed, with equal variances among habitats. The following results were obtained.

Answer: see next page.

Mean (number of fledglings):
Forests: 6.1 Wetlands: 7.1 Grasslands: 4.0

Source	SS	DF	MS
Total	250.0	N-1 = 30-1 = 29	
Among habitats	50.0	k-1 = 3-1 = 2	Groups SS/Groups DF = 50.0/2 = 25.0
Within habitats	TotSS-GrSS=200.0	N-k = 30-3 = 27	Error SS/Error DF = 200.0/27 = 7.41

a) State the null hypothesis appropriate to address the research question using the study described above.

H_0: $\mu_{\text{forest}} = \mu_{\text{wetland}} = \mu_{\text{grassland}}$

b) Complete the table above. Show both numerical values and formulas used.

See answers in table above.

c) State the criterion for rejection of the null hypothesis, including both symbolic and numerical expressions for the test statistic (e.g., criterion for a t-test might be: $t_{\alpha(2),\nu} = t_{0.05(2),60} = 2.000$).

$F_{calc} \geq F_{crit}$ $\quad F_{crit} = F_{\alpha(1),k-1,N-k} = F_{0.05(1),2,27} = 3.35$

d) Present a complete statistical analysis to test the hypothesis that nest productivity does not differ among the three habitats. Include a conclusion using words analogous to those in the research question.

$$F_{calc} = \frac{GroupsMS}{errorMS} = \frac{25.0}{7.41} = 3.375$$

Since $F_{calc} > F_{crit}$ ($F_{0.05(1),2,27} = 3.35$), reject H_0.

Conclude that means are not all equal.

Word Conc.: Productivity differs among songbird nests in forests, wetlands, and grasslands.

9 Suppose analysis for problem 8 showed that mean nest productivity is not equal in forests, wetlands, and grasslands. Which habitats differ in mean productivity? Use a significance level of $\alpha\,0.05$.

Tukey test for multiple comparisons

$H_0 : \mu_A = \mu_B$, for A,B $\in$ {forests, wetlands, grasslands}

Critical value: $q_{0.05,\nu,k} = q_{0.05,27,3} = 3.532$

$$q = \frac{\overline{X}_B - \overline{X}_A}{SE} \qquad \text{SE} = \sqrt{\frac{\text{error MS}}{n}} = \sqrt{\frac{7.41}{10}} = 0.86$$

1st compare wetland nests vs. grassland nests:

$$q = \frac{7.1 - 4.0}{0.86} = 3.6$$

reject H_0

Next compare forest nests vs. grassland nests:

$$q = \frac{6.1 - 4.0}{0.86} = 2.44$$

do not reject H_0

No need to compare smaller differences (i.e., forests vs. wetlands).

Conclude: productivity of wetland nests is greater than productivity of grassland nests.

10 Research question: Does DDE in the tissues of birds cause them to lay thin eggs? You decide to address this question using simple linear regression, comparing eggshell thickness with DDE concentration in tissue samples from 100 thrushes. You obtain the following result.

a (intercept) = 30.0 $s_a = 3.45$
b (slope) = -2.8 $s_b = 0.96$

Source of variation	DF	SS	MS
Total	99	875.44	
Regression	1	224.31	224.31
Residual	98	651.13	6.644

a) Present a complete statistical analysis to test the hypothesis that there is no relationship between eggshell thickness in thrushes and DDE concentration in their tissues (i.e., that the slope of the regression line equals zero).

Note regression ANOVA table completed above.

t-test: (suppose $\alpha = 0.05$)

$$t = \frac{b - \beta_0}{s_b} = \frac{-2.8 - 0.0}{0.96} = -2.917$$

e.g., for $\alpha = 0.05$, $t_{0.05(2),98} = 1.984$

$t\_calc > t\_crit$, $0.0025 > P > 0.001$ => reject H_o

Conclude that DDE causes thinning in (thrush) eggshells, at rate of -2.8 units per unit of DDE.

Note: because *Ho* is one-tailed, must use *t*-test, above. For your review, below is how you could test the analogous two-tailed hypothesis using an *F*-test.

F-test:

$$F = \frac{\text{regression MS}}{\text{residual MS}} = 33.76$$

e.g., for $\alpha = 0.05$, $F_{\alpha(1),1,n-2} = F_{0.05(1),1,98} = 3.95$ (use 90 DF because 98 not in table => 90)

$F_{\text{calc}} > F_{\text{crit}}$, $P << 0.0005$ => reject H_o

b) Write the equation that relates tissue DDE concentration (X) with eggshell thickness (Y).

$$Y_i = \alpha + \beta X_i + \varepsilon_i$$

c) What fraction of total variation in initial eggshell thickness is explained by regression results above?

$$r^2 = \frac{\text{regression SS}}{\text{total SS}} = 224.31 / 875.44 = 0.256$$

d) Predict the thickness of shells laid by a thrush whose tissues contain DDE at 4.4 ppm.

$$Y_i = 30.0 - 2.8X_i$$
$$Y_i = 30.0 - 2.8 * 4.4 = 17.7$$

11 You are trying to determine whether yellow warblers are migrating to breeding areas earlier in spring than they did two decades ago. (Yellow warblers are small songbirds that winter in Central America and breed throughout North America.) You have data on yellow warbler spring arrival dates two decades ago at twenty locations. If you record dates of yellow warbler arrival at those same twenty locations this spring, what is the minimum advance in arrival date you could detect with 90% power, at a significance level of 0.05? Your estimate of the variance in arrival date differences is 20.0 days^2.

Minimum advance (difference, then – now) detectable @ 90% power; paired-sample t-test. One-tailed test (*advance* in arrival date), with ν = 20-1=19.

$$\delta = \sqrt{\frac{s_d^2}{n}}(t_{\alpha,\nu} + t_{\beta(1),\nu})$$

$$\delta = \sqrt{\frac{s_d^2}{n}}(t_{0.05(1),19} + t_{0.10(1),19})$$

$$\delta = \sqrt{\frac{20.0}{20}}(1.729 + 1.328)$$

$$\delta = (1.0)(3.057)$$

$$\delta = 3.057$$

Minimum advance in arrival date detectable is 3 days.