Two-Sample Hypothesis Testing

1 Test for difference between two Means (2-sample t-test)

1.1 Hypotheses: H_0: $\mu_1 = \mu_2$, alternatively: H_0: $\mu_1 - \mu_2 = 0$,
H_A: $\mu_1 \neq \mu_2$ H_A: $\mu_1 - \mu_2 \neq 0$

1.2 Assumptions
- 1.2.1 both samples from normal populations
- 1.2.2 equal population variances

1.3 $$t = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1 - \bar{X}_2}}$$

- $s_{\bar{X}_1 - \bar{X}_2}$ SE of difference betw/ means
- t depends on ν, degrees of freedom
  - $\nu = \nu_1 + \nu_2$ or $\nu = n_1 + n_2 - 2$
- reject H_0 if $|t_{calc}| \geq t_{\alpha(2),\nu}$ (t_α from Zar table B3)

1.4 Calculating $s_{\bar{X}_1 - \bar{X}_2}$
- var (difference between independent variables) = sum of individual variances,

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} \quad \rightarrow \text{assume equal variances} \quad \sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}$$

- estimate σ^2; use both s_1^2 & s_2^2: $$s_p^2 = \frac{SS_1 + SS_2}{\nu_1 + \nu_2}$$
- so: $$s^2_{\bar{X}_1 - \bar{X}_2} = \frac{s_p^2}{n_1} + \frac{s_p^2}{n_2} \quad \text{and} \quad s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$
- finally, $$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

1.5 One-Tailed test Hypotheses:

H_0: $\mu_1 \geq \mu_2$, H_A: $\mu_1 < \mu_2$; if $t \leq -t_{\alpha(1),\nu}$ then reject H_0

H_0: $\mu_1 \leq \mu_2$, H_A: $\mu_1 > \mu_2$; if $t \geq t_{\alpha(1),\nu}$ then reject H_0

1.6 Violations of Two-Sample t-test Assumptions
- 1.6.1 normal distribution: t-test is very robust (one-tailed test sensitive to skew)
- 1.6.2 equal variances: if unequal, greater chance of type I error (greater than α)
  correction: Welch's approximate t

$$t' = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \text{and d.f.} \quad \nu' = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

– often, ν' not integer; use next smallest integer (e.g., if $\nu' = 8.75$, use $\nu = 8$)

 (continued)

2 Confidence Limits for Population Means

2.1 $1-\alpha$ Confidence interval for pop *i*: $\bar{X}_i \pm t_{\alpha(2),\nu}\sqrt{\dfrac{s_p^2}{n_i}}$

2.2 If unequal variances, $\bar{X}_i \pm t_{\alpha(2),\nu}\sqrt{\dfrac{s_i^2}{n_i}}$

2.3 Confidence limits for difference betw means: $\bar{X}_i - \bar{X}_2 \pm t_{\alpha(2),\nu} s_{\bar{X}_1-\bar{X}_2}$

2.4 if H_0 not rejected (samples from pops w/ identical means, μ):

- estimate of μ is weighted average of sample means: $\bar{X}_p = \dfrac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2}$
- $1-\alpha$ CI for μ: $\bar{X}_p \pm t_{\alpha(2),\nu}\sqrt{\dfrac{s_p^2}{n_i+n_2}}$

3 Testing for Difference betw Variances

3.1 Null Hypothesis (2-tailed): H_0: $\sigma_1^2 = \sigma_2^2$; H_A: $\sigma_1^2 \neq \sigma_2^2$

3.2 Variance ratio test

$F = \dfrac{s_1^2}{s_2^2}$ or $F = \dfrac{s_2^2}{s_1^2}$, whichever larger; i.e., larger sample variance in numerator

3.3 *F*-distribution: Zar, Table B4

- $F_{\alpha,\nu 1,\nu 2}$ order of ν_1, ν_2 matters; $F_{\alpha,\text{numerator df, denominator df}}$
- if H_0 not rejected, best estimate of σ^2 is pooled variance, s_p^2

3.4 Variance ratio test severely affected by non-normality

4 Paired-Sample t-test

4.1 Paired data

4.2 Hypotheses:

2-tailed: H_0: $\mu_d = 0$, H_A: $\mu_d \neq 0$

1-tailed:

H_0: $\mu_d \geq \mu_0$, H_A: $\mu_d < \mu_0$ or: H_0: $\mu_d \leq \mu_0$, H_A: $\mu_d > \mu_0$

4.3 Equivalent to a one-sample test

d_i = difference betw/ paired measurements

sample size = # pairs of data degrees freedom = *n*-1

4.4 Test Statistic:

$t = \dfrac{\bar{d}}{s_{\bar{d}}}$ $\bar{d}$ = mean difference

$s_{\bar{d}}$ = standard error of mean difference

4.5 Assumption: differences, d_i, are normally distributed

→ not need assumptions from 2-sample *t*-test (normality & equality of variances)

5 Confidence Limits for Mean Difference

5.1 similar procedure as with one-sample t-test;

5.2 $1-\alpha$ confidence limits for μ_d: $\bar{d} \pm t_{\alpha(2),\nu} s_{\bar{d}}$