Statistical Power

1 Sample size needed for given width of confidence interval

1.1 Desired confidence interval → how large sample size needed?

1.2 Suppose CI: $\overline{X} \pm d$, d = ½ width of CI;

- $d = t_{\alpha(2),\nu} s_{\overline{X}}$
- sample size needed:

$$n = \frac{s^2}{d^2} t^2_{\alpha(2),(n-1)} \qquad \text{(Zar, Eq. 7.7)}$$

- reliability of estimate of n: depends on accuracy of approximation of σ^2 by s^2

2 Types of Errors

2.1 Type I error: incorrect rejection of a true null hypothesis
(= say is different when really is not)

2.2 Type II error: failure to reject false null hypothesis
(= say is not different when really is)

2.3 Two other (correct) outcomes: (1) accept true H_0; (2) reject false H_0

3 Power of one-sample t-tests

3.1 Power of a test = probability of rejecting the null hypothesis when it is false
Power = 1- β

3 related ?'s: (1) sample size needed for given power
(2) Minimum difference detectable @ given power
(3) Power of test for given situation

3.2 Power depends on 4 factors:
(1) significance level (α)
(2) sample size (n)
(3) difference between μ_0 and true μ (=δ)
(4) sample variance, s^2

3.3 Three ways to increase power:
(1) increase α (usually inadvisable, but sometimes warranted)
(2) increase sample size
(3) different statistical test

3.4 Sample size needed:

$$n = \frac{s^2}{\delta^2}(t_{\alpha,\upsilon} + t_{\beta(1),\upsilon})^2 \qquad \text{(Zar, Eq. 7.8)}$$

- Problem: ν depends on n => cannot solve explicitly
→ iterate, converge on solution

(continued)

3.5 Minimum difference that can be detected @ given power

- from Eq. 7.8, solve for δ: $$\delta = \sqrt{\frac{s^2}{n}}(t_{\alpha,\nu} + t_{\beta(1),\nu})$$
- n specified; can solve for δ explicitly

3.6 Power of a test for a given situation: α, n, δ.

- from Eq. 7.8, solve for t_β: $$t_{\beta(1),\nu} = \frac{\delta}{\sqrt{\frac{s^2}{n}}} - t_{\alpha,\nu}$$
- solve explicitly (approximately; not all critical values of t-distribution given)

→ if approximate $t_{\beta(1),\nu}$ w/ normal variable, $Z_{\beta(1)}$ then can determine β

4 Power of two-sample t-tests

4.1 Sample size needed:

$$n = \frac{2s_p^2}{\delta^2}(t_{\alpha,\upsilon} + t_{\beta(1),\upsilon})^2$$ (Zar, Eq. 8.22)

- Problem: ν depends on n => cannot solve explicitly

→ iterate, converge on solution

4.2 Minimum difference that can be detected @ given power

- from Eq. 8.22, solve for δ: $$\delta = \sqrt{\frac{2s_p^2}{n}}(t_{\alpha,\nu} + t_{\beta(1),\nu})$$
- n specified; can solve for δ explicitly

4.3 Power of a test for a given situation: α, n, δ.

- from Eq. 8.22, solve for t_β: $$t_{\beta(1),\nu} = \frac{\delta}{\sqrt{\frac{2s_p^2}{n}}} - t_{\alpha,\nu}$$
- solve explicitly (approximately; not all critical values of *t*-distribution given)

→ if approximate $t_{\beta(1),\nu}$ w/ normal variable, $Z_{\beta(1)}$ then can determine β

5 For more information on statistical power, see Zar.

If you want to explore power further, see the following wwweb sites:

http://davidmlane.com/hyperstat/power.html (and links therein)
http://www.psycho.uni-duesseldorf.de/aap/projects/gpower/how_to_use_gpower.html
http://statpages.org/
http://www.mp1-pwrc.usgs.gov/powcase/index.html
http://www.mp1-pwrc.usgs.gov/powcase/primer.html
http://www.mp1-pwrc.usgs.gov/powcase/steps.html

Links to free power analysis packages:

http://statpages.org/#Power
http://www.mp1-pwrc.usgs.gov/powcase/monitor.html
http://www.psycho.uni-duesseldorf.de/aap/projects/gpower/
http://hotspur.psych.yorku.ca/SCS/Online/power/index.html