1 Data Transformations

1.1 Parametric test assumptions

1.2 What to do when assumptions violated?
- nonparametric test
- transform data; parametric analysis on transformed values

1.3 General procedure for data transformation
    1.3.1 raw data variable: X
    1.3.2 transformed variable: $X' = f(X)$
- choose transformation function f so X' meets assumptions of parametric test

    1.3.3 perform statistical test using transformed data, X'
        e.g., two-sample t-test:

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}} \; ; \quad \text{use: } t = \frac{\overline{X}_1' - \overline{X}_2'}{s_{\overline{X}_1' - \overline{X}_2'}}$$

    1.3.4 invert transformation to obtain mean, etc.

2 Logarithmic transformation

2.1 Applications:
- ANOVA: when factor effects are multiplicative (not additive)
- when heteroscedastic & SD proportional to mean
- regression: transform independent (X) variable when proportional (e.g., 2x or 10x) change in X causes small (linear) change in dependent variable (Y)
- regression: transform both variables when "true" relationship is a power law, e.g., Basal Metabolic Rate = a (body mass)^b, where a, b constants

2.2 Transformation function:
$X' = \log X$ generally use: $X' = \log(X + 1)$

2.3 If X' normally distributed, then X is lognormal.

3 Square Root transformation

3.1 Applications: heteroscedastic samples; when group variances proportional to means

3.2 Transformation function:
$X' = \sqrt{X}$ generally use: $X' = \sqrt{X + \frac{1}{2}}$

4 Arcsine transformation

4.1 Applications: variables that are proportions or %'s,
- converts binomial distribution to nearly normal

4.2 Transformation function: $p' = \arcsin\sqrt{p}$

4.3 Inverse transformation: $p = (\sin p')^2$

5 Reciprocal transformation

5.1 Applications: •ANOVA: if group SD $\propto$ (group $\overline{X}$)^2 •Regression: data on hyperbolic curve

5.2 Transformation function: $X' = \dfrac{1}{X}$ or $X' = \dfrac{1}{X+1}$

6 Square (power) transformation

6.1 Applications: • if SD decrease as means increase   • if distribution is skewed left

6.2 Transformation function: $X' = X^2$