Age-Structured Populations

I. **Notation**

A. Age: x = <u>age</u> of individual (yr) $x \in [0,k]$ newborn x =0 oldest age x = k
B. Age class: (age class x: age betw/ x & x+1)
C. Relationship betw/ ages (x) & age classes(i)
<u>Notational convention</u>: age in parentheses, age class as subscript
e.g., l(3) = P{survival to start of 3rd year} l_2 = P{survival thru 2nd age class} l(3) = l_2

II. **Fertility & survivorship schedules**

A. Maternity function (plot of fertility rate)
- b(x) or m(x) = average # offspring/female @ age x
- area under curve = # offspring betw 1st & 2nd yrs (e.g., betw/ 20 & 30)

B. Survivorship function
- S(x) = # surviving to age x
- suvivorship function: l(x) = proportion surviving to year x
  l(x) = S(x) / S(0)
- age specific survival rate = P{live to x+1 | x} = $g(x) = \dfrac{l(x+1)}{l(x)}$
- examples: types I, II, III

C. <u>Note</u>: l(x) = survival up to start of age x; b(x), m(x) = per capita fertility @ age x

D. Population pyramid: age structure; combine l(x) & m(x)

III. **Model Assumptions**: similar to basic exptl growth – except w/ age structure

IV. **Net Reproductive Rate, R_o & Generation Time**

A. Def.: net reproductive rate, R_o, = mean # offspring / female over lifetime
- multiply P{survival to age x} * fecundity @ age x
- sum across all ages $$R_0 = \sum_{x=0}^{k} l(x)b(x)$$
- if R_o = 1: Zero Pop Growth
  if R_o > 1: exponential growth
  if R_o < 1: exponential decline

B. Def: generation time, G = average age of production

$$G = \frac{\sum_{x=0}^{k} x l(x)b(x)}{\sum_{x=0}^{k} l(x)b(x)}$$

C. Intrinsic rate of increase (r):
- Euler eq: $$1 = \sum_{x=0}^{k} e^{-rx} l(x)b(x)$$
- solve iteratively (numerically); start w/ approximation: $r \approx \dfrac{\ln(R_0)}{G}$

(continued)

V. **Leslie Matrix**

A. Describes Δ's in N due to mortality & reproduction

convert l(x) & b(x) to age-specific survival & fecundity

Always of form:

$$\mathbf{A} = \begin{matrix} F1 & F2 & F3 & F4 \\ P1 & 0 & 0 & 0 \\ 0 & P2 & 0 & 0 \\ 0 & 0 & P3 & 0 \end{matrix}$$

each entry = transition / change # individuals from age class to another

N(t+1) = **AN**(t)

B. 4 Main ideas of Linear Algebra:

1. vector space: every vector can be represented as combination of basis vectors
2. Matricies are operators in vector space: 3 kinds
   a) stretch & contract: zeros in off- diagonal
   b) rotate vectors $\begin{pmatrix} 0.87 & -0.5 \\ 0.5 & 0.87 \end{pmatrix}$ → rotates ccw 30°
   c) reduces to form $\begin{pmatrix} \lambda & 0 \\ 1 & \lambda \end{pmatrix}$
3. Free to choose coordinate frame (basis vectors)
4. w/ appropriate choice of coord frame, all matricies reduce to one of 3 kinds

C. Application to Leslie matrix & Age Structured populations

- dominant eigenvalue = growth rate
- eigenvector = stable age distribution

VI. **Stable Age Distribution**

- Rapid convergence on Stable Age Distribution,
  – time to convergence: as many time steps as there are age classes
- Proportion in each age class, c(x) = # in each age class ÷ total pop size

$$c(x) = \frac{e^{-rx} l(x)}{\sum_{x=0}^{k} e^{-rx} l(x)}$$

VII. **Implications for Life Histories**

A. Tradeoffs: survival & reprod @ various ages
  – w/in physiological constraints

B. Senescence (evolution of)

VIII. **Demographic Momentum**

A. Example: Age Structure in Sweden vs. Costa Rica

B. Problem of pop control & demographic momentum in India

C. Demographic predictions of human pop: assumptions?