Metapopulation Processes

Patch Occupancy Models

I. **Isolated populations**: $P_x = 1 - (p_e)^x$

x = # patches
P = P{extinction/yr) of all x pops
p^e = P{extinction/patch}

II. **General Patch Occupancy Model**

$$\frac{df}{dt} = I - E$$

III. **Mainland - Island or Reservoir - Satellite** (simple case)

$$I = p_i(1 - f) \qquad E = p_e f$$

$$\frac{df}{dt} = p_i(1 - f) - p_e f$$

$$\hat{f} = \frac{p_i}{p_i + p_e}$$

$\hat{f}$ = steady state fraction occupied patches

- persistence guaranteed

Assumptions:

- – homogeneous patches
- – patches identical
- – p_e & p_i constant
- – no time lags

IV. **Internal colonization; no mainland**

$$p_i = if \qquad E = p_e f$$

i = effect of occupied patches on P{colonization}

$$\frac{df}{dt} = if(1 - f) - p_e f$$

$$\hat{f} = 1 - \frac{p_e}{i}$$

- persistence not guaranteed (need balance i vs. e)

V. **Mainland - island; w/propagule rain & rescue effect**

$$I = p_i(1 - f) \qquad E = p_e f \qquad p_e = e(1 - f)$$

e = p_e w/out rescue effect

$$\frac{df}{dt} = p_i(1 - f) - ef(1 - f)$$

$$\hat{f} = \frac{p_i}{e}$$

- persistence guaranteed (mainlaind-island)

VI. **Internal colonization w/ rescue effect**

$$\frac{df}{dt} = if(1 - f) - ef(1 - f)$$

set df/dt = 0; solve for $\hat{f} \rightarrow (0,1,f)$

$\hat{f} = 0 \quad i < e$; $\hat{f} = 1 \quad i > e$; $\hat{f} = f_o \quad i = e$

VII. **Metapopulation with habitat loss**

Finite number of patches (c.f. infinite number of patches assumed in models above)

Only h fraction of f patches are suitable (habitat loss = 1 – h)

$$\frac{df}{dt} = chf(1-f) - ef$$

c, e = colonization, extinction rate parameters

$$\hat{f} = 1 - \frac{e/c}{h} \quad \text{or} \quad \hat{f} = 1 - \frac{\delta}{h}, \quad \delta = e/c$$

$\hat{f} > 0$ only if $\delta < h$

Note: $\hat{f}$ is the equilibrium fraction of suitable patches occupied (i.e., $\hat{f}$ out of h possible)

δ is threshold amount of habitat required for persistence;

If destroy enough habitat so that $h < \delta$, then metapopulation (regional) extinction.

Metapopulation Potential

Hanski I and O Ovaskainen. 2000. The metapopulation capacity of a fragmented landscape. *Nature* 404:755-758.

Finite number of unique patches (i); patch-specific probabilities of occupancy: $p_i(t)$.

Rate of change in occupancy probability:

$$\frac{dp_i(t)}{dt} = (\text{Colonization rate}_\text{i})[1 - p_i(t)] - (\text{Extinction rate}_\text{i})p_i(t)$$

For example, (particular functional forms)

$$\text{Colonization rate}_\text{i} = c\sum_{j \neq i} \exp(-\alpha d_{ij}) A_j p_j(t)$$

$$\text{Extinction rate}_\text{i} = e / A_i$$

c, e = constants

A_i = area of patch i

d_{ij} = distance betw/ patches i, j

$1/\alpha$ = mean migration distance

Matrix notation:

Metapopulation matrix **M**, with elements m:

$$m_{ii} = 0$$

$$m_{ij} = A_i A_j \exp(-\alpha d_{ij}), \quad j \neq i$$

Equilibrium solution, with $\hat{p}_i > 0$, iff

$$\lambda_M > \delta$$

λ_M is leading eigenvalue of **M**

Condition for persistence in given landscape.

Weighted average of occupancy probabilities:

$$\hat{p}_\lambda = 1 - \frac{\delta}{\lambda_M}$$

Note: λ_M same functional role as h in model VII.

Contribution of patch i to metapopulation potential (λ_M):

$\lambda_i = x_i^2 \lambda_M$, where x_i is i^{th} element in the leading eigenvector of matrix **M**

Source-Sink Dynamics

A. habitat 1 = source; $\bar{\lambda} > 1$ habitat 2 = sink; $\bar{\lambda} < 1$ recall: $\bar{\lambda} = (\lambda_1 \lambda_2 \ldots \lambda_t)^{1/t}$

B. Model population change thru time
- n_1 grows to n_1* (= max density supportable in habitat 1)
- @ max density, $\bar{\lambda}_1 n_1^* = n_1^* + n_1^*(\bar{\lambda}_1 - 1)$
  => annual influx of $n_1^*(\bar{\lambda}_1 - 1)$ from habitat 1
- habitat 2 reaches equilibrium: $n_2^* = n_1^* \left(\dfrac{\lambda_1 - 1}{1 - \lambda_2} \right)$ $\lambda_1 - 1$ = reproductive surplus in source
  $1 - \lambda_2$ = reprod. deficit in sink

C. Conclusion: higher N in sink than in source, if surplus in source >. deficit in sink
- "Most of the individuals in a local population may exist in habitat that cannot maintain the population." – Pulliam 1988 *Am.Nat*

Quantitative Incidence Function Model & applications to *Melitaea cinxia*

A. Incidence of pop in patch "i":

(1) $$J_i = \frac{C_i}{C_i + E_i}$$ J_i = P{patch i occupied}

C_i = P{ patch i colonized}
E_i = P{ extinction in patch i}

B. Extinction Probabilities: Assume $E_i \propto$ Area of patch I (because $E_i \propto N_i$)

(2) $$E_i = \min\left[\frac{\mu}{A_i^x}, 1 \right]$$ A_i = area of patch I μ, x: fitted parameters

C. Colonization Probabilities = f(patch area & locations of extant populations)

(3) $$C_i = \frac{M_i^2}{M_i^2 + y^2}$$ M_i = # migrants to patch i per year

(4) $$M_i = \beta \sum_{j=1}^{n} p_j e^{-\alpha d_{ij}} A_j = \beta s_i$$ p_j = 1 if patch j occupied, 0 if empty

d_{ij} = distance between patched i & j

y, α, β = fitted parameters

D. Rescue effect: assume patches can be rescued from extinction by immigration:

(5) $$J_i = \frac{1}{1 + \dfrac{\mu'}{s_i^2 A_i^x}}$$ $\mu' = \mu y'^2$; $y' = y / \beta$ for patches > A_0, critical area

($A_i < A_0 \Rightarrow E_i = 1$)

- **Essential point**: incidence function (eq.1) is transformed into parameterized model
  – can be fitted to empirical occupancy data
  – calculation of "metapopulation capacity" & persistence threshold

(Hanski & Ovaskainen 2000. Nature 404:755-758.